

ERUVIO MENTORING · EXTERNAL-STYLE EXAMINATION

Mathematical Methods

Units 1 and 2 · Algebra, statistics and functions; calculus and further functions

Paper 1 · Section 2

SHORT RESPONSE · 45 MARKS

STUDENT DETAILS

GIVEN NAME(S)

FAMILY NAME

SCHOOL

DATE

BOOK OF

DO NOT WRITE ON THIS PAGE

Structure of this examination

PAPER	SECTION	QUESTION TYPE	TIME	MARKS
Paper 1 TECHNOLOGY-FREE	Section 1	Multiple choice, 10 questions	5 min perusal	10
	Section 2	Short response, 9 questions	90 min working	45
Paper 2 TECHNOLOGY-ACTIVE	Section 1	Multiple choice, 10 questions	5 min perusal	10
	Section 2	Short response, 9 questions	90 min working	45
Total		38 questions	190 minutes	110

Degree of difficulty

Every question in this examination is tagged with its degree of difficulty. The tag appears in brass beside the marks.

DEGREE OF DIFFICULTY	MARKS	PROPORTION
Simple familiar SF	66	60%
Complex familiar CF	22	20%
Complex unfamiliar CU	22	20%
Total	110	100%

Instructions

- **Perusal time: 5 minutes. Working time: 90 minutes.**
- **This is the technology-free paper.** Calculators, notes and the formula book may not be used.
- Write using black or blue pen. Planning paper will not be marked.
- All questions are drawn from Unit 1 (algebra, statistics and functions) and Unit 2 (calculus and further functions).
- **Each question is tagged with its degree of difficulty** in brass beside the marks: **SF** simple familiar, **CF** complex familiar, **CU** complex unfamiliar.
- **Section 2 contains 9 short response questions.** Answer all questions in the spaces provided. Show all working.

Show all working. Questions worth more than one mark require mathematical reasoning or working to be shown to support the answer.

Paper 1 · Section 2

Short response · 9 questions · 45 marks · technology-free

1 An arithmetic sequence has first term 7 and fifth term 23. [4 marks]

(a) Determine the common difference. [2] SF

WORKING

$d =$ _____

(b) Determine the sum of the first twenty terms. [2] SF

WORKING

$S_{20} =$ _____

2 Consider the function $y = x^4 - 3x^2 + 5x$. [4 marks]

(a) Determine dy/dx . [2] SF

WORKING

$dy/dx =$ _____

(b) Determine the gradient of the curve at $x = 2$. [2] SF

WORKING

Gradient = _____

3 This question is about indices and logarithms. [5 marks]

(a) Solve $2^{x+1} = 8^x - 1$.

[2] **SF**

WORKING

$x =$ _____

(b) Evaluate $\log_5(125)$.

[1] **SF**

WORKING

Value = _____

(c) Solve $\log_3(x) + \log_3(x - 2) = 1$.

[2] **CF**

WORKING

$x =$ _____

4 A box contains 5 white balls and 3 black balls. Two balls are drawn at random without replacement. [5 marks]

(a) Determine the probability that both balls are white.

[2] **SF**

WORKING

Probability = _____

(b) Determine the probability that exactly one of the two balls is black.

[3] **CF**

WORKING

Probability = _____

5 This question is about quadratic functions.

[5 marks]

(a) Express $x^2 - 6x + 11$ in the form $(x + p)^2 + q$.

[2] **SF**

WORKING

Expression = _____

(b) Determine all values of k for which $x^2 + kx + 9 = 0$ has exactly one solution.

[3] **CF**

WORKING

$k =$ _____

6 This question is about trigonometric functions.

[5 marks]

(a) Determine the exact value of $\cos(5\pi/6)$.

[2] **SF**

WORKING

Value = _____

(b) Solve $2 \cos(x) = 1$ for $0 \leq x \leq 2\pi$.

[3] **CF**

WORKING

$x =$ _____

7 Consider the function $f(x) = x^3 - 6x^2 + 9x$.

[6 marks]

(a) Determine $f'(x)$ and hence find the x coordinates of the stationary points.

[2] **SF**

WORKING

$x =$ _____

- (b) Determine the coordinates of each stationary point and classify it as a local maximum or a local minimum.

[2] **CF**

WORKING

Local maximum _____ Local minimum _____

- (c) Determine all values of c for which the equation $f(x) = c$ has exactly three distinct real solutions. Justify your answer.

[2] **CU**

WORKING

8 Consider the parabola $y = x^2 - 3x$.

[6 marks]

- (a) Determine the equation of the tangent to the parabola at $x = 1$.

[3] **SF**

WORKING

Equation = _____

(b) The line $y = ax - 4$ is a tangent to this parabola. Determine all possible values of a .

[3] CU

WORKING

$a =$ _____

9 The sum of the first n terms of a sequence is given by $S_n = 2n^2 + 3n$. Determine the tenth term of the sequence, and prove that the sequence is arithmetic.

[5] CU

WORKING

$t_{10} =$ _____

END OF PAPER