

ERUVIO MENTORING · EXTERNAL-STYLE EXAMINATION

Mathematical Methods

Units 1 and 2 · Algebra, statistics and functions; calculus and further functions

Paper 2 · Section 2

SHORT RESPONSE · 45 MARKS

STUDENT DETAILS

GIVEN NAME(S)

FAMILY NAME

SCHOOL

DATE

BOOK OF

DO NOT WRITE ON THIS PAGE

Structure of this examination

PAPER	SECTION	QUESTION TYPE	TIME	MARKS
Paper 1 TECHNOLOGY-FREE	Section 1	Multiple choice, 10 questions	5 min perusal	10
	Section 2	Short response, 9 questions	90 min working	45
Paper 2 TECHNOLOGY-ACTIVE	Section 1	Multiple choice, 10 questions	5 min perusal	10
	Section 2	Short response, 9 questions	90 min working	45
Total		38 questions	190 minutes	110

Degree of difficulty

Every question in this examination is tagged with its degree of difficulty. The tag appears in brass beside the marks.

DEGREE OF DIFFICULTY	MARKS	PROPORTION
Simple familiar SF	66	60%
Complex familiar CF	22	20%
Complex unfamiliar CU	22	20%
Total	110	100%

Instructions

- **Perusal time: 5 minutes. Working time: 90 minutes.**
- **This is the technology-active paper.** A QCAA-approved graphics calculator or CAS may be used, and the QCAA formula book is provided.
- Write using black or blue pen. Planning paper will not be marked.
- All questions are drawn from Unit 1 (algebra, statistics and functions) and Unit 2 (calculus and further functions).
- **Each question is tagged with its degree of difficulty** in brass beside the marks: **SF** simple familiar, **CF** complex familiar, **CU** complex unfamiliar.
- **Section 2 contains 9 short response questions.** Answer all questions in the spaces provided. Show all working.

Show all working. Questions worth more than one mark require mathematical reasoning or working to be shown to support the answer.

Paper 2 · Section 2

Short response · 9 questions · 45 marks · technology-active

- 1 A geometric sequence has first term 800 and fourth term 100. [4 marks]

(a) Determine the common ratio. [2] SF

WORKING

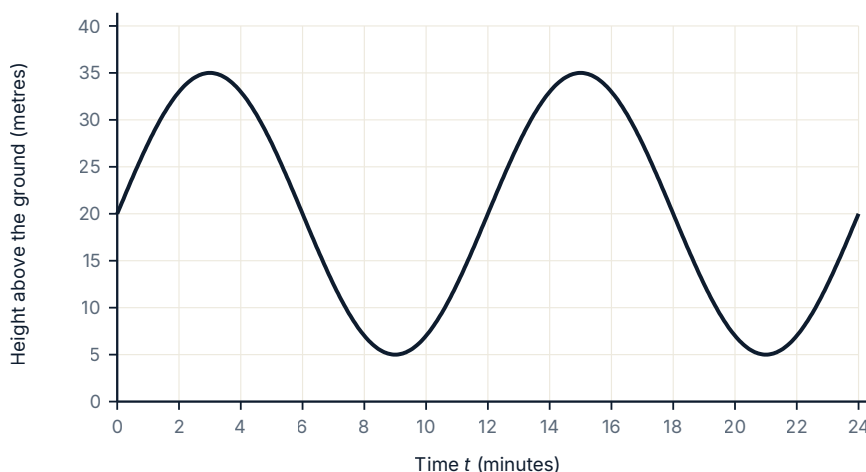
$r =$ _____

(b) Determine the sum to infinity of the sequence. [2] SF

WORKING

$S_{\infty} =$ _____

- 2 A seat on a Ferris wheel is at a height of h metres above the ground after t minutes, where $h = 20 + 15 \sin(\pi t/6)$. The graph of this function is shown below. [4 marks]



(a) State the maximum and the minimum height of the seat above the ground. [2] SF

Maximum = _____ m Minimum = _____ m

(b) Determine the period of the motion and explain what it represents.

[2] SF

WORKING

3 The value of a machine, in dollars, after t years is given by $V = 4000(0.88)^t$.

[5 marks]

(a) Determine the value of the machine after 5 years, correct to the nearest dollar.

[2] SF

WORKING

$V = \$$ _____

(b) Determine, correct to two decimal places, the time taken for the value of the machine to fall to \$2000.

[3] SF

WORKING

$t =$ _____ years

4 The total cost, in dollars, of producing x units of a product is $C(x) = 0.02x^3 - 1.5x^2 + 40x + 500$.

[5 marks]

(a) Determine $C'(x)$ and evaluate $C'(10)$.

[3] **SF**

WORKING

$C'(x) =$ _____ $C'(10) =$ _____

(b) Determine the production level x at which the marginal cost $C'(x)$ is a minimum.

[2] **CF**

WORKING

$x =$ _____

5 The two-way table below shows data collected from 100 students about whether they play a sport and whether they play a musical instrument. [5 marks]

	Plays music	Does not play music	Total
Plays sport	18	27	45
Does not play sport	22	33	55
Total	40	60	100

(a) A student is chosen at random. Determine the probability that the student plays a sport and plays a musical instrument.

[2] **SF**

WORKING

Probability = _____

- (b) Determine the probability that a student plays a musical instrument given that the student plays a sport, and hence decide whether the two events are independent.

[3] **SF**

WORKING

- 6 Consider $f(x) = x^3 - 9x^2 + 24x - 16$.

[5 marks]

- (a) Show that $x = 1$ is a zero of f , and hence write $f(x)$ in fully factorised form.

[3] **SF**

WORKING

$f(x) =$ _____

- (b) Determine the coordinates of the stationary points of the curve.

[2] **CF**

WORKING

Stationary points = _____

- 7 A population of fish in a lake is modelled by $N = 1200/(1 + 29e^{-0.4t})$, where N is the number of fish after t years.

[6 marks]

(a) Determine the initial population and the population after 10 years.

[2] SF

WORKING

$N(0) =$ _____ $N(10) =$ _____

(b) Determine the time taken for the population to reach 600 fish, and explain what happens to the population in the long term. Justify your answer using the model.

[4] CU

WORKING

8 A discrete random variable X has the probability distribution shown below, where a and b are constants. The expected value of X is 3.0.

[6 marks]

x	1	2	3	4
$P(X = x)$	0.1	0.2	a	b

(a) Determine the values of a and b .

[3] CF

WORKING

$a =$ _____ $b =$ _____

(b) Determine the variance and the standard deviation of X .

[3] CU

WORKING

Variance = _____ Standard deviation = _____

9 An open box is made from a rectangular sheet of cardboard measuring 20 cm by 12 cm by cutting a square of side x cm from each corner and folding up the sides. Determine the value of x that gives the largest possible volume, and state that volume. Give both answers correct to two decimal places.

[5] CU

WORKING

$x = \rule{1cm}{0.4pt}$ cm $\rule{10cm}{0.4pt}$ Volume = $\rule{1cm}{0.4pt}$ cm³ $\rule{10cm}{0.4pt}$

END OF PAPER