

ERUVIO MENTORING · EXTERNAL-STYLE EXAMINATION

Mathematical Methods

Units 3 and 4 · Further calculus; further functions and statistics

Paper 1 · Section 2

SHORT RESPONSE · 45 MARKS

STUDENT DETAILS

GIVEN NAME(S)

FAMILY NAME

SCHOOL

DATE

BOOK OF

DO NOT WRITE ON THIS PAGE

Structure of this examination

PAPER	SECTION	QUESTION TYPE	TIME	MARKS
Paper 1 TECHNOLOGY-FREE	Section 1	Multiple choice, 10 questions	5 min perusal	10
	Section 2	Short response, 9 questions	90 min working	45
Paper 2 TECHNOLOGY-ACTIVE	Section 1	Multiple choice, 10 questions	5 min perusal	10
	Section 2	Short response, 9 questions	90 min working	45
Total		38 questions	190 minutes	110

Degree of difficulty

Every question in this examination is tagged with its degree of difficulty. The tag appears in brass beside the marks.

DEGREE OF DIFFICULTY	MARKS	PROPORTION
Simple familiar SF	66	60%
Complex familiar CF	22	20%
Complex unfamiliar CU	22	20%
Total	110	100%

Instructions

- **Perusal time: 5 minutes. Working time: 90 minutes.**
- **This is the technology-free paper.** Calculators, notes and the formula book may not be used.
- Write using black or blue pen. Planning paper will not be marked.
- All questions are drawn from Unit 3 (further calculus) and Unit 4 (further functions and statistics).
- **Each question is tagged with its degree of difficulty** in brass beside the marks: **SF** simple familiar, **CF** complex familiar, **CU** complex unfamiliar.
- **Section 2 contains 9 short response questions.** Answer all questions in the spaces provided. Show all working.

Show all working. Questions worth more than one mark require mathematical reasoning or working to be shown to support the answer.

Paper 1 · Section 2

Short response · 9 questions · 45 marks · technology-free

1 Differentiate each of the following with respect to x .

[4 marks]

(a) $y = x^2 \ln(x)$

[2] **SF**

WORKING

$\frac{dy}{dx} = \underline{\hspace{2cm}}$

(b) $y = (3x - 1)^5$

[2] **SF**

WORKING

$\frac{dy}{dx} = \underline{\hspace{2cm}}$

2 Evaluate each of the following integrals.

[4 marks]

(a) $\int (6x^2 - 4x + 3) \, dx$

[2] **SF**

WORKING

Integral = $\underline{\hspace{2cm}}$

(b) $\int_1^2 2/x \, dx$, giving an exact answer

[2] **SF**

WORKING

Integral = _____

3 This question is about exponential functions.

[5 marks]

(a) Determine dy/dx for $y = e^{2x} \sin(x)$.

[3] **SF**

WORKING

$dy/dx =$ _____

(b) Solve $e^{2x} - 5e^x + 6 = 0$, giving exact answers.

[2] **CF**

WORKING

$x =$ _____

4 Consider the curves $y = 4 - x^2$ and $y = x + 2$.

[5 marks]

(a) Evaluate $\int_0^2 (4 - x^2) dx$.

[2] **SF**

WORKING

Value = _____

(b) Determine the exact area of the region enclosed between the two curves.

[3] **CF**

WORKING

Area = _____

5 This question is about trigonometric functions.

[5 marks]

(a) Determine dy/dx for $y = 3 \sin(2x) - \cos(x)$.

[2] **SF**

WORKING

$dy/dx =$ _____

(b) Solve $2 \sin(x) \cos(x) = \sin(x)$ for $0 \leq x \leq 2\pi$.

[3] **CF**

WORKING

$x =$ _____

6 A fair coin is tossed four times. Let X be the number of heads obtained, so that X follows a binomial distribution with $n = 4$ and $p = 1/2$.

[5 marks]

(a) Determine $P(X = 2)$, giving an exact answer.

[2] **SF**

WORKING

Probability = _____

(b) Determine $P(X \geq 3)$ and the expected value of X .

[3] **CF**

WORKING

$P(X \geq 3) =$ _____ $E(X) =$ _____

7 Consider $f(x) = x^3 - 3x^2 + 2$.

[6 marks]

(a) Determine the coordinates of the stationary points of the curve.

[2] **SF**

WORKING

Stationary points = _____

(b) Determine the coordinates of the point of inflection.

[2] **CF**

WORKING

Point of inflection = _____

(c) Determine all values of k for which $f(x) = k$ has exactly three distinct real solutions. Justify your answer.

[2] **CU**

WORKING

8 A rectangle has one side on the x axis and two vertices on the curve $y = 12 - x^2$, one in the first quadrant at $(x, 12 - x^2)$ and one at $(-x, 12 - x^2)$, where $0 < x < 2\sqrt{3}$.

[6 marks]

(a) Show that the area of the rectangle is $A = 24x - 2x^3$, and determine dA/dx .

[3] **SF**

WORKING

(b) Determine the exact value of x that maximises the area, justify that it is a maximum, and state the maximum area.

[3] **CU**

WORKING

$x =$ _____ $A =$ _____

9 A curve has gradient function $dy/dx = 3x^2 + k$, where k is a constant. The tangent to the curve at $x = 1$ has gradient 7, and the curve passes through the point $(0, 2)$. Determine the equation of the curve, and hence determine $\int_0^2 y \, dx$.

[5] **CU**

WORKING

$y =$ _____ $\text{Integral} =$ _____

END OF PAPER