

ERUVIO MENTORING · EXTERNAL-STYLE EXAMINATION

Mathematical Methods

Units 3 and 4 · Further calculus; further functions and statistics

Paper 2 · Section 2

SHORT RESPONSE · 45 MARKS

STUDENT DETAILS

GIVEN NAME(S)

FAMILY NAME

SCHOOL

DATE

BOOK OF

DO NOT WRITE ON THIS PAGE

Structure of this examination

PAPER	SECTION	QUESTION TYPE	TIME	MARKS
Paper 1 TECHNOLOGY-FREE	Section 1	Multiple choice, 10 questions	5 min perusal	10
	Section 2	Short response, 9 questions	90 min working	45
Paper 2 TECHNOLOGY-ACTIVE	Section 1	Multiple choice, 10 questions	5 min perusal	10
	Section 2	Short response, 9 questions	90 min working	45
Total		38 questions	190 minutes	110

Degree of difficulty

Every question in this examination is tagged with its degree of difficulty. The tag appears in brass beside the marks.

DEGREE OF DIFFICULTY	MARKS	PROPORTION
Simple familiar SF	66	60%
Complex familiar CF	22	20%
Complex unfamiliar CU	22	20%
Total	110	100%

Instructions

- **Perusal time: 5 minutes. Working time: 90 minutes.**
- **This is the technology-active paper.** A QCAA-approved graphics calculator or CAS may be used, and the QCAA formula book is provided.
- Write using black or blue pen. Planning paper will not be marked.
- All questions are drawn from Unit 3 (further calculus) and Unit 4 (further functions and statistics).
- **Each question is tagged with its degree of difficulty** in brass beside the marks: **SF** simple familiar, **CF** complex familiar, **CU** complex unfamiliar.
- **Section 2 contains 9 short response questions.** Answer all questions in the spaces provided. Show all working.

Show all working. Questions worth more than one mark require mathematical reasoning or working to be shown to support the answer.

Paper 2 · Section 2

Short response · 9 questions · 45 marks · technology-active

- 1 The mass of a bag of sugar is normally distributed with a mean of 500 g and a standard deviation of 50 g. Give all probabilities correct to four decimal places. [4 marks]

(a) Determine the probability that a randomly chosen bag has a mass less than 560 g. [2] SF

WORKING

Probability = _____

(b) Determine the probability that a randomly chosen bag has a mass between 450 g and 550 g. [2] SF

WORKING

Probability = _____

- 2 A random variable X follows a binomial distribution with $n = 12$ and $p = 0.25$. [4 marks]

(a) Determine $P(X = 3)$, correct to four decimal places. [2] SF

WORKING

Probability = _____

(b) Determine the mean and the standard deviation of X .

[2] SF

WORKING

Mean = _____ Standard deviation = _____

3 Evaluate each of the following.

[5 marks]

(a) $\int_1^4 (2x + 1/\sqrt{x}) \, dx$, giving an exact answer

[2] SF

WORKING

Value = _____

(b) The area under the curve $y = e^{0.5x}$ between $x = 0$ and $x = 2$, correct to four decimal places

[3] SF

WORKING

Area = _____

4 In a random sample of 400 voters, 180 said they support a proposed change.

[5 marks]

- (a) Determine the sample proportion and an approximate 95% confidence interval for the population proportion. Give the endpoints correct to four decimal places.

[3] **SF**

WORKING

Interval = _____

- (b) A campaigner claims that a majority of all voters support the change. Decide whether the confidence interval supports this claim, and justify your answer.

[2] **CF**

- 5 A closed cylindrical can is to hold 500 cm^3 . Its radius is r cm and its height is h cm.

[5 marks]

- (a) Show that the total surface area is $S = 2\pi r^2 + 1000/r$.

[2] **SF**

WORKING

- (b) Determine the value of r that minimises the surface area, and the minimum surface area. Give both correct to two decimal places.

[3] **SF**

WORKING

$r = \underline{\hspace{2cm}} \text{ cm}$ $S = \underline{\hspace{2cm}} \text{ cm}^2$

- 6 The temperature in a greenhouse, in degrees Celsius, is modelled by $T = 18 + 6 \sin((\pi(t - 8))/12)$, where t is the number of hours after midnight and $0 \leq t \leq 24$. [5 marks]

- (a) State the maximum and minimum temperatures, and determine the time of day at which the maximum occurs. [3] **SF**

WORKING

Maximum = $\underline{\hspace{2cm}}$ Minimum = $\underline{\hspace{2cm}}$ Time = $\underline{\hspace{2cm}}$

- (b) Determine the values of t for which the temperature exceeds 21°C . [2] **CF**

WORKING

Interval = $\underline{\hspace{2cm}}$

- 7 A particle moves in a straight line with velocity $v = 3t^2 - 12t + 9$ metres per second, where t is the time in seconds. The particle starts at the origin. [6 marks]

(a) Determine the times at which the particle is instantaneously at rest.

[2] **SF**

WORKING

$t = \text{_____ s}$ _____

(b) Determine the total distance travelled by the particle during the first 4 seconds. Justify your method.

[4] **CU**

WORKING

Distance = _____ m _____

8 A machine fills bags of flour. The mass of a filled bag is normally distributed with standard deviation 15 g. A bag is rejected if its mass is below 1000 g.

[6 marks]

(a) The machine is set so that 2% of bags are rejected. Determine the mean mass, correct to one decimal place.

[3] **CF**

WORKING

$\mu = \text{_____ g}$ _____

- (b) The manufacturer wants no more than 1% of bags rejected, but does not want to increase the mean beyond the value found in part (a). Determine the largest standard deviation that would achieve this, correct to two decimal places, and comment on what the machine would need to do.

[3] CU

WORKING

$\sigma = \text{----- g}$ _____

- 9 A chemical dissolves so that the mass remaining, m grams, satisfies $dm/dt = -0.4m$. Initially 50 g is present. Verify that $m = 50e^{-0.4t}$ satisfies both the differential equation and the initial condition, determine the time at which 10 g remains, and determine the rate at which the mass is decreasing at that instant.

[5] CU

WORKING

$t = \text{_____ min}$ _____ $\text{Rate} = \text{_____ g min}^{-1}$ _____

END OF PAPER