ERUVIO MENTORING · EXTERNAL-STYLE EXAMINATION

Specialist Mathematics

Units 1 and 2 · Combinatorics, proof, vectors and matrices; complex numbers and trigonometry

Paper 1 · Section 2

SHORT RESPONSE · 55 MARKS

STUDENT DETAILS

GIVEN NAME(S) _____ FAMILY NAME _____

SCHOOL _____ DATE _____ BOOK OF _____

DO NOT WRITE ON THIS PAGE

Structure of this examination

PAPER	SECTION	QUESTION TYPE	TIME	MARKS
Paper 1 TECHNOLOGY-FREE	Section 1	Multiple choice, 10 questions	5 min perusal 90 min working	10
	Section 2	Short response, 9 questions		55
Paper 2 TECHNOLOGY-ACTIVE	Section 1	Multiple choice, 10 questions	5 min perusal 90 min working	10
	Section 2	Short response, 9 questions		55
Total		**38 questions**	**190 minutes**	**130**

Degree of difficulty

Every question in this examination is tagged with its degree of difficulty. The tag appears in brass beside the marks.

DEGREE OF DIFFICULTY	MARKS	PROPORTION
Simple familiar SF	78	60%
Complex familiar CF	26	20%
Complex unfamiliar CU	26	20%
Total	**130**	**100%**

Instructions

- **Perusal time: 5 minutes. Working time: 90 minutes.**
- **This is the technology-free paper.** Calculators, notes and the formula book may not be used.
- Write using black or blue pen. Planning paper will not be marked.
- All questions are drawn from Unit 1 (combinatorics, proof, vectors and matrices) and Unit 2 (complex numbers, trigonometry, functions and matrices).
- **Each question is tagged with its degree of difficulty** in brass beside the marks: SF simple familiar, CF complex familiar, CU complex unfamiliar.
- **Section 2 contains 9 short response questions.** Answer all questions in the spaces provided. Show all working.

Show all working. Questions worth more than one mark require mathematical reasoning or working to be shown to support the answer.

Paper 1 · Section 2

Short response · 9 questions · 55 marks · technology-free

1 This question is about counting. [5 marks]

(a) Evaluate ${}^{7}C_{3}$ and ${}^{7}P_{3}$. [2] SF

WORKING

${}^{7}C_{3}$ = _____ ${}^{7}P_{3}$ = _____

(b) Determine the number of distinct arrangements of the letters of the word PARALLEL. [3] SF

WORKING

Number = _____

2 Let $\mathbf{a} = 3\boldsymbol{i} + 4\boldsymbol{j}$ and $\mathbf{b} = -\boldsymbol{i} + 2\boldsymbol{j}$. [5 marks]

(a) Determine $|\mathbf{a}|$ and $\mathbf{a} + 2\mathbf{b}$. [2] SF

WORKING

$|\mathbf{a}|$ = _____ $\mathbf{a} + 2\mathbf{b}$ = _____

(b) Determine $\mathbf{a} \cdot \mathbf{b}$ and hence the exact value of the cosine of the angle between $\mathbf{a}$ and $\mathbf{b}$. [3] SF

WORKING

$\mathbf{a} \cdot \mathbf{b}$ = _____ cos θ = _____

3 Let $z = 2 + 3i$ and $w = 1 - i$. [6 marks]

(a) Determine $z + w$ and zw. [2] SF

WORKING

$z + w$ = _____ zw = _____

(b) Express z/w in the form $a + bi$. [2] SF

WORKING

Answer = _____

(c) Determine all complex numbers $a + bi$, where a and b are real, such that $(a + bi)^2 = 5 + 12i$. [2] CF

WORKING

Answer = _____

4 Let $A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$. [6 marks]

(a) Determine AB. [2] SF

WORKING

AB = _____

(b) Determine det(A) and A^{-1}. [2] SF

WORKING

det(A) = _____ A^{-1} = _____

(c) Hence solve the system $2x + y = 5$ and $3x + 4y = 10$. [2] SF

WORKING

$x =$ _____ $y =$ _____

5 This question is about trigonometry. [6 marks]

(a) Determine the exact values of $\sin(7\pi/6)$ and $\cos(7\pi/6)$. [2] SF

WORKING

sin = _____ cos = _____

(b) Solve $2\sin(x) = \sqrt{3}$ for $0 \leq x \leq 2\pi$. [2] SF

WORKING

$x =$ _____

(c) Prove the identity $(1 - \cos 2\theta)/(\sin 2\theta) = \tan \theta$. [2] CF

WORKING

6 The points A and B have position vectors $\mathbf{OA} = 2\boldsymbol{i} + \boldsymbol{j}$ and $\mathbf{OB} = 4\boldsymbol{i} + 5\boldsymbol{j}$. [6 marks]

(a) Determine **AB** and its exact magnitude. [3] SF

WORKING

AB = _____ |**AB**| = _____

(b) The point P divides AB internally in the ratio 1 : 3. Determine the position vector of P. [3] CF

WORKING

OP = _____

7 This question is about proof. [7 marks]

(a) Prove directly that the sum of any two even integers is even. [3] SF

WORKING

(b) Prove by contradiction that $\sqrt{2}$ is irrational. [4] CU

WORKING

8 Let $M = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and $N = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$. [7 marks]

(a) Describe geometrically the transformation represented by each of M and N, determine NM, and describe the single transformation that NM represents. [5] CF

WORKING

(b) Determine the matrix that maps (1, 0) to (3, 1) and (0, 1) to (−1, 2), and state what its determinant tells you about areas under this transformation. [2] CU

WORKING

Matrix = _____

9 In the Argand plane, $z_1 = 1 + i$ and $z_2 = 3 + 2i$ are adjacent vertices of a square, labelled anticlockwise. Determine the other two vertices, verify that the four points form a square, and determine its area. *You may use the fact that multiplication by i rotates a vector through 90° anticlockwise.* [7] CU

WORKING

Vertices = _____ Area = _____

END OF PAPER