

ERUVIO MENTORING · EXTERNAL-STYLE EXAMINATION

Specialist Mathematics

Units 1 and 2 · Combinatorics, proof, vectors and matrices; complex numbers and trigonometry

Paper 2 · Section 2

SHORT RESPONSE · 55 MARKS

STUDENT DETAILS

GIVEN NAME(S)

FAMILY NAME

SCHOOL

DATE

BOOK OF

DO NOT WRITE ON THIS PAGE

Structure of this examination

PAPER	SECTION	QUESTION TYPE	TIME	MARKS
Paper 1 TECHNOLOGY-FREE	Section 1	Multiple choice, 10 questions	5 min perusal	10
	Section 2	Short response, 9 questions	90 min working	55
Paper 2 TECHNOLOGY-ACTIVE	Section 1	Multiple choice, 10 questions	5 min perusal	10
	Section 2	Short response, 9 questions	90 min working	55
Total		38 questions	190 minutes	130

Degree of difficulty

Every question in this examination is tagged with its degree of difficulty. The tag appears in brass beside the marks.

DEGREE OF DIFFICULTY	MARKS	PROPORTION
Simple familiar SF	78	60%
Complex familiar CF	26	20%
Complex unfamiliar CU	26	20%
Total	130	100%

Instructions

- **Perusal time: 5 minutes. Working time: 90 minutes.**
- **This is the technology-active paper.** A QCAA-approved graphics calculator or CAS may be used, and the QCAA formula book is provided.
- Write using black or blue pen. Planning paper will not be marked.
- All questions are drawn from Unit 1 (combinatorics, proof, vectors and matrices) and Unit 2 (complex numbers, trigonometry, functions and matrices).
- **Each question is tagged with its degree of difficulty** in brass beside the marks: **SF** simple familiar, **CF** complex familiar, **CU** complex unfamiliar.
- **Section 2 contains 9 short response questions.** Answer all questions in the spaces provided. Show all working.

Show all working. Questions worth more than one mark require mathematical reasoning or working to be shown to support the answer.

Paper 2 · Section 2

Short response · 9 questions · 55 marks · technology-active

- 1 A password consists of four different letters chosen from the 26 letters of the alphabet, followed by two different digits. [5 marks]

(a) Determine the total number of possible passwords. [2] SF

WORKING

Number = _____

- (b) Determine how many of these passwords contain no vowels. Treat A, E, I, O and U as the vowels. [3] SF

WORKING

Number = _____

- 2 Let $\mathbf{a} = 5\mathbf{i} - 2\mathbf{j}$ and $\mathbf{b} = 3\mathbf{i} + 4\mathbf{j}$. [5 marks]

(a) Determine the angle between $\mathbf{a}$ and $\mathbf{b}$, correct to two decimal places. [2] SF

WORKING

$\theta =$ _____

(b) Determine the scalar projection and the vector projection of **a** onto **b**.

[3] **SF**

WORKING

Scalar projection = _____ Vector projection = _____

3 Let $z = -2 + 2\sqrt{3}i$.

[6 marks]

(a) Determine the modulus and the argument of z .

[2] **SF**

WORKING

$|z| =$ _____ $\arg(z) =$ _____

(b) Write z in polar form.

[2] **SF**

WORKING

$z =$ _____

(c) Hence determine z^3 , giving your answer in the form $a + bi$.

[2] **CF**

WORKING

$z^3 =$ _____

4 Consider the system of simultaneous equations below.

[6 marks]

$$x + y + z = 4 \quad 2x - y + z = 8 \quad x + 2y - z = -3$$

(a) Write the system as a matrix equation and determine the determinant of the coefficient matrix.

[3] **SF**

WORKING

Determinant = _____

(b) Solve the system.

[3] **SF**

WORKING

$x =$ _____ $y =$ _____ $z =$ _____

5 The depth of water at a jetty, in metres, is modelled by $d = 6 + 2.5 \sin(\pi t/6)$, where t is the number of hours after midnight.

[6 marks]

(a) State the maximum and minimum depths and the period of the model.

[2] **SF**

Maximum = _____ m _____ Minimum = _____ m _____ Period = _____ h _____

(b) Determine the depth at $t = 4$, correct to two decimal places.

[2] **SF**

WORKING

$d =$ _____ m _____

(c) A vessel needs a depth of more than 7.5 m. Determine the times during the first 12 hours for which it can use the jetty.

[2] **CF**

WORKING

Interval = _____

6 A boat is steered on a bearing of 040° at a speed of 12 km h^{-1} relative to the water. A current of 5 km h^{-1} flows due east.

[6 marks]

(a) Determine the east and north components of the resultant velocity, and hence the resultant speed, correct to two decimal places.

[3] **SF**

WORKING

East = _____ North = _____ Speed = _____

- (b) Determine the true bearing along which the boat actually travels, and the time taken to cover 30 km along that track. Give the bearing to the nearest degree.

[3] **CF**

WORKING

Bearing = _____ Time = _____ h _____

- 7 A committee of 4 people is chosen at random from a group of 6 women and 4 men.

[7 marks]

- (a) Determine the total number of possible committees, and the number containing exactly 2 women.

[3] **SF**

WORKING

Total = _____ Exactly 2 women = _____

- (b) Determine the probability that the committee contains at least 2 women, giving an exact answer. Explain why calculating this directly is more efficient than using the complement in this case.

[4] CU

WORKING

8 Let $M = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and $N = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$.

[7 marks]

- (a) Describe the transformation represented by each matrix, determine NM , and describe the single transformation represented by NM . State the effect of NM on the area of a shape.

[5] CF

WORKING

- (b) Determine the matrix that represents a reflection in the line $y = 2x$, and verify your matrix by applying it to a point on the line.

[2] CU

WORKING

Matrix = _____

- 9 OABC is a rhombus, so that all four sides have equal length. Let $\mathbf{OA} = \mathbf{a}$ and $\mathbf{OC} = \mathbf{c}$. Using vector methods only, prove that the diagonals OB and AC are perpendicular.

[7] CU

WORKING

END OF PAPER