ERUVIO MENTORING · EXTERNAL-STYLE EXAMINATION

Specialist Mathematics

Units 3 and 4 · Induction, vectors, matrices and complex numbers; integration, differential equations and statistical inference

Paper 1 · Section 2

SHORT RESPONSE · 55 MARKS

STUDENT DETAILS

GIVEN NAME(S) _____ FAMILY NAME _____

SCHOOL _____ DATE _____ BOOK OF _____

DO NOT WRITE ON THIS PAGE

Structure of this examination

PAPER	SECTION	QUESTION TYPE	TIME	MARKS
Paper 1 TECHNOLOGY-FREE	Section 1	Multiple choice, 10 questions	5 min perusal 90 min working	10
	Section 2	Short response, 9 questions		55
Paper 2 TECHNOLOGY-ACTIVE	Section 1	Multiple choice, 10 questions	5 min perusal 90 min working	10
	Section 2	Short response, 9 questions		55
Total		**38 questions**	**190 minutes**	**130**

Degree of difficulty

Every question in this examination is tagged with its degree of difficulty. The tag appears in brass beside the marks.

DEGREE OF DIFFICULTY	MARKS	PROPORTION
Simple familiar SF	78	60%
Complex familiar CF	26	20%
Complex unfamiliar CU	26	20%
Total	**130**	**100%**

Instructions

- **Perusal time: 5 minutes. Working time: 90 minutes.**
- **This is the technology-free paper.** Calculators, notes and the formula book may not be used.
- Write using black or blue pen. Planning paper will not be marked.
- All questions are drawn from Unit 3 (mathematical induction, and further vectors, matrices and complex numbers) and Unit 4 (integration, differential equations and statistical inference).
- **Each question is tagged with its degree of difficulty** in brass beside the marks: SF simple familiar, CF complex familiar, CU complex unfamiliar.
- **Section 2 contains 9 short response questions.** Answer all questions in the spaces provided. Show all working.

Show all working. Questions worth more than one mark require mathematical reasoning or working to be shown to support the answer.

Paper 1 · Section 2

Short response · 9 questions · 55 marks · technology-free

1 Let $\mathbf{a} = \boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}$ and $\mathbf{b} = 2\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}$. [5 marks]

(a) Determine $\mathbf{a} \cdot \mathbf{b}$ and $|\mathbf{a}|$. [2] SF

WORKING

$\mathbf{a} \cdot \mathbf{b}$ = _____ $|\mathbf{a}|$ = _____

(b) Determine $\mathbf{a} \times \mathbf{b}$, and verify that it is perpendicular to $\mathbf{a}$. [3] SF

WORKING

$\mathbf{a} \times \mathbf{b}$ = _____

2 Evaluate each of the following integrals. [5 marks]

(a) $\int 2x(x^2 + 1)^4 \, dx$ [2] SF

WORKING

Integral = _____

(b) $\int_0^{\pi/2} \sin(x)\cos(x)\,dx$ [3] SF

WORKING

Value = _____

3 Let $z = 1 + \sqrt{3}\,i$. [6 marks]

(a) Determine the modulus and the argument of z. [2] SF

WORKING

$|z|$ = _____ arg(z) = _____

(b) Use De Moivre's theorem to determine z^6. [2] SF

WORKING

z^6 = _____

(c) Determine all solutions of $z^3 = 8$, giving exact answers in the form $a + bi$. [2] CF

WORKING

Solutions = _____

4 A quantity y satisfies $dy/dx = xy$, with $y = 2$ when $x = 0$. [6 marks]

(a) Solve the differential equation by separating the variables. [4] SF

WORKING

y = _____

(b) Determine the exact value of y when $x = 2$. [2] SF

WORKING

y = _____

5 Consider $(3x + 1)/((x + 1)(x - 2))$. [6 marks]

(a) Express the expression in partial fractions and hence determine its indefinite integral. [4] SF

WORKING

Integral = _____

(b) Evaluate the definite integral from $x = 3$ to $x = 4$, giving an exact answer. [2] CF

WORKING

Value = _____

6 A line passes through the point A(1, 2, 3) and has direction vector $2\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$. [6 marks]

(a) Write a vector equation for the line, and determine whether the point B(5, 0, 7) lies on it. [3] SF

WORKING

(b) A second line has direction vector $\boldsymbol{i} + 2\boldsymbol{j} + 2\boldsymbol{k}$. Determine the exact value of the cosine of the acute angle between the two lines. [3] CF

WORKING

cos θ = _____

7 This question is about proof by mathematical induction. Let $S(n)$ be the statement [7 marks]

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = (n(n+1)(2n+1))/6$$

(a) Verify that $S(n)$ holds for n = 1, 2 and 3. [3] SF

WORKING

(b) Prove by mathematical induction that $S(n)$ holds for all positive integers n. [4] CU

WORKING

8 Consider the curves $y = x^2$ and $y = x^3$ for $0 \leq x \leq 1$. [7 marks]

(a) Determine the exact area of the region enclosed between the two curves, and the exact volume generated when the region under $y = x^2$ alone is rotated about the x axis. [5] CF

WORKING

Area = _____ Volume = _____

(b) Determine the exact volume generated when the region *between* the two curves is rotated about the x axis. [2] CU

WORKING

Volume = _____

9 Determine all solutions of $z^4 = -16$, giving exact answers in the form $a + bi$. Show that the four solutions are the vertices of a square in the Argand plane, and determine the exact area of that square. [7] CU

WORKING

Solutions = _____ Area = _____

END OF PAPER