

ERUVIO MENTORING · EXTERNAL-STYLE EXAMINATION

Specialist Mathematics

Units 3 and 4 · Induction, vectors, matrices and complex numbers; integration, differential equations and statistical inference

Paper 2 · Section 2

SHORT RESPONSE · 55 MARKS

STUDENT DETAILS

GIVEN NAME(S)

FAMILY NAME

SCHOOL

DATE

BOOK OF

DO NOT WRITE ON THIS PAGE

Structure of this examination

PAPER	SECTION	QUESTION TYPE	TIME	MARKS
Paper 1 TECHNOLOGY-FREE	Section 1	Multiple choice, 10 questions	5 min perusal	10
	Section 2	Short response, 9 questions	90 min working	55
Paper 2 TECHNOLOGY-ACTIVE	Section 1	Multiple choice, 10 questions	5 min perusal	10
	Section 2	Short response, 9 questions	90 min working	55
Total		38 questions	190 minutes	130

Degree of difficulty

Every question in this examination is tagged with its degree of difficulty. The tag appears in brass beside the marks.

DEGREE OF DIFFICULTY	MARKS	PROPORTION
Simple familiar SF	78	60%
Complex familiar CF	26	20%
Complex unfamiliar CU	26	20%
Total	130	100%

Instructions

- **Perusal time: 5 minutes. Working time: 90 minutes.**
- **This is the technology-active paper.** A QCAA-approved graphics calculator or CAS may be used, and the QCAA formula book is provided.
- Write using black or blue pen. Planning paper will not be marked.
- All questions are drawn from Unit 3 (mathematical induction, and further vectors, matrices and complex numbers) and Unit 4 (integration, differential equations and statistical inference).
- **Each question is tagged with its degree of difficulty** in brass beside the marks: **SF** simple familiar, **CF** complex familiar, **CU** complex unfamiliar.
- **Section 2 contains 9 short response questions.** Answer all questions in the spaces provided. Show all working.

Show all working. Questions worth more than one mark require mathematical reasoning or working to be shown to support the answer.

Paper 2 · Section 2

Short response · 9 questions · 55 marks · technology-active

- 1 A random sample of 36 components has a mean lifetime of 52.4 hours. The population standard deviation is known to be 6 hours. [5 marks]

(a) Determine the standard deviation of the distribution of sample means. [2] SF

WORKING

Value = _____

- (b) Determine an approximate 95% confidence interval for the population mean, giving the endpoints correct to two decimal places. [3] SF

WORKING

Interval = _____

- 2 Let $\mathbf{a} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ and $\mathbf{b} = \mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$. [5 marks]

(a) Determine the angle between $\mathbf{a}$ and $\mathbf{b}$, correct to two decimal places. [2] SF

WORKING

$\theta =$ _____

(b) Determine $\mathbf{a} \times \mathbf{b}$ and its magnitude, and state what that magnitude represents geometrically.

[3] **SF**

WORKING

$\mathbf{a} \times \mathbf{b} =$ _____ Magnitude = _____

3 The region bounded by $y = x^2 + 1$, the x axis, $x = 0$ and $x = 2$ is rotated about the x axis.

[6 marks]

(a) Write the definite integral that gives the volume of the solid of revolution, and evaluate it exactly.

[4] **SF**

WORKING

Volume = _____

(b) Determine the volume correct to two decimal places, and state the units.

[2] **CF**

WORKING

Volume = _____

- 4 An object at 90°C is placed in a room held at 20°C . Its temperature T satisfies $\frac{dT}{dt} = -k(T - 20)$, where t is in minutes. After 10 minutes the object has cooled to 60°C . [6 marks]

(a) Show by separating the variables that $T = 20 + 70e^{-kt}$. [3] SF

WORKING

(b) Determine k correct to four decimal places, and hence the temperature after 25 minutes. [3] SF

WORKING

$k = \underline{\hspace{1cm}}$ $T(25) = \underline{\hspace{1cm}}^\circ\text{C}$

- 5 This question is about the cube roots of $27i$. [6 marks]

(a) Express $27i$ in polar form. [2] SF

WORKING

Polar form = $\underline{\hspace{1cm}}$

(b) Determine all three cube roots of $27i$ in polar form.

[2] **SF**

WORKING

Roots = _____

(c) Express the root with the smallest positive argument in the form $a + bi$, giving exact values.

[2] **CF**

WORKING

Root = _____

6 Consider the curves $y = \sin(x)$ and $y = \cos(x)$ for $\pi/4 \leq x \leq 5\pi/4$.

[6 marks]

(a) Determine the exact area of the region enclosed between the two curves.

[3] **SF**

WORKING

Area = _____

- (b) Explain why the integrand must be taken as $\sin(x) - \cos(x)$ rather than the other way round over this interval.

[3] **CF**

- 7 A quality control engineer measures the fill volume of bottles. The population standard deviation is known to be 8 mL.

[7 marks]

- (a) A sample of 64 bottles has a mean of 501.2 mL. Determine an approximate 95% confidence interval for the population mean, correct to two decimal places.

[3] **SF**

WORKING

Interval = _____

- (b) The engineer wants the margin of error to be no more than 1.5 mL. Determine the smallest sample size that achieves this, and explain why quadrupling the sample size only halves the margin of error.

[4] CU

WORKING

$n =$ _____

- 8 A population P , in thousands, satisfies $dP/dt = 0.5P(1 - P/8)$, where t is measured in years and $P = 1$ when $t = 0$.

[7 marks]

- (a) Determine the values of P for which the population is stationary, and determine the value of P at which the population is growing fastest. Justify your answers.

[5] **CF**

WORKING

- (b) Without solving the differential equation, sketch the shape of the graph of P against t for $t \geq 0$, and describe the long-term behaviour.

[2] **CU**

SKETCH

- 9 The point P has coordinates $(4, 1, 2)$. A line has vector equation $\mathbf{r} = (\mathbf{i} + 3\mathbf{k}) + t(2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$. Determine the shortest distance from P to the line, and the coordinates of the point on the line closest to P . Verify that the vector joining P to that point is perpendicular to the line.

[7] **CU**

WORKING

Distance = _____ Closest point = _____

END OF PAPER